What teaching means to me…

Sound mathematical reasoning is vital to success in the STEM disciplines. As a mathematics teacher, my primary goal is for my students to discover the power and importance of rigorous deductive reasoning. If there's one thing I want my students to retain long after a course has finished, it's that complex questions should be methodically dissected and known results should be applied with precision. Mathematics, with its unique blend of the general and the specific, is the perfect environment to hone these universally applicable skills.

There are many challenges in making this goal a reality. The first is personal. Mathematics is a discipline that many of us come to with rigid preconceived notions. Some have have always loved it, whereas others are more conflicted. Negative experiences can contribute to a deep anxiety about the subject, and a fixed mindset about one's ability to progress. To add to this, many people identify as members of groups that are stereotyped as being less inclined to succeed in STEM disciplines, seriously undermining mathematical confidence.

The second challenge is structural. Mathematics is a constructive subject: to progress, one must have mastered what comes before. Serious omissions in a student's foundational knowledge can catastrophically impact their progress.

These two issues make effective mathematical tuition a challenging and complex task. This is especially true in lower division university courses, where hundreds of students with a diverse spectrum of knowledge and experience are brought together in one class. To compound matters, the academic importance of such courses cannot be overstated. If a student has negative experiences in such classes, it may drastically affect later academic success.

Having taught mathematics at the undergraduate and graduate level for over a decade, and to over 10 000 students, I've come to believe that there is no single answer as to what constitutes truly effective mathematics tuition. Rather, it is the combination of many key principles, working together to reinforce each other.

The following are the central principles around which I structure all my teaching:

- Accessible, Clear, and Highly Organized Exposition

  In class, the exposition should be focused and organized with a consistent, logical structure. It should blend the formal with the informal, combining crisp, precise statements with more intuitive explanations. Key points and important conclusions should be visibly highlighted. I also like to emphasize visual reasoning, making sure students have a strong intuitive understanding of potentially confusing technical details.

Structurally, the general theory should be continuously reinforced with well chosen examples, which should be worked out in detail in class by the instructor. All relevant computational steps should be included, with key subtleties and common misunderstandings emphasized. This is especially crucial in large lower division classes, where the instructor must engage with a broad spectrum abilities. Ideally, in these less sophisticated classes one should introduce the general theory through appropriate special cases.

I also feel that a strong emphasis should be placed on systematic, methodical reasoning. When possible, complex processes should be summarized with diagrams (Appendix A) to give the students a succinct and easy to navigate overview.

In class, I believe the exposition should be neatly handwritten in real time. This both helps to slow the pace and allows the instructor to share their own thought process step-by-step.

- Energy and Enthusiasm

  Learning mathematics can be a transformative experience. It can open doors like few other subjects and this makes it exciting for both the student and the teacher. As an instructor, I always try to encourage my students' sense of wonder and discovery.

  When unexpected things happen, like the emergence of the Mandelbrot set or the relationship between the structure of DNA and ordinary differential equations, I can't help but share my own amazement. For students who have only ever seen mathematics as a dry computation subjected, such experiences can help illuminate the path they may take in the future.

- Active and Passive Learning

  Traditionally, university mathematics is taught in a formal lecture environment, where students absorb information passively. While this approach is important and indeed, highly effective for some, when used exclusively, it can leave many students feeling disconnected from the subject.

  There is clear evidence that students learn more effectively when they are actively engaging with the material and are being provided with real-time feedback. While there are challenges to using such techniques in large classes, there are effective ways to blend them with traditional methods. After introducing a new concept, I pose carefully prepared questions, designed to allow students to discover hidden subtleties themselves. I encourage the class to attempt them in small groups, while I and a

teaching assistant walk around providing support. Afterwards, we all discuss the most effective approaches and common misunderstandings.

- Teaching Resources Outside the Classroom

  While engaging, informative lectures are vital to good instruction, it's important to understand that much of the process happens outside of the classroom. As such, I believe it's important to provide students with easily accessible, comprehensive learning resources on a central website.

  On my own course websites I provide topic by topic, personally recorded video lectures, giving students the opportunity to revisit concepts at their own pace. This also allows me to use visual and audio aids that wouldn't be possible at a chalkboard. I also provide complete lecture notes, practice exams and homework solutions. Everyone learns a little differently, and this gives my students the flexibility to approach the class in a way that works best for them.

- Organized Course Management, Clear Expectations and Transparency

  All aspects of any course should be meticulously organized, from the punctual updating of online resources to the efficient and consistent grading of exams.

  From the outset, the class should clearly understand the practical aspects of the class and the instructor expectations. In the classes I teach, on my website I provide a comprehensive course policy giving information about all aspects of the course, from grading policies to practice exams. Having a centralized location for all course matters grounds the course and gives the class the stability to focus on learning.

- Support, Approachability and Availability

  While giving students the resources to succeed is a necessary part of effective mathematics instruction, it's not sufficient. I feel that the instructor needs to show they genuinely care about their students, both as learners and people. What does this mean in practice?

  To care about students as learners means to put yourself in their shoes: to understand their mathematical perspective and more broadly, the role mathematics will play throughout their academic lives. This understanding should inform how each topic is presented. Knowing where students are coming from, and what they are hoping to achieve, is essential to being able to describe difficult concepts in an engaging way with both clarity and depth.

To care about students as people means recognizing the barriers that keep them from doing their best and to be flexible when external factors disrupt their lives. Essential to this is being generous with your time as an instructor and to be as available as possible.

Office hours are a key part of this, and I hold them for two hours every weekday, often attracting over fifty participants. If students are struggling outside of class, I try to direct them to relevant university resources. If they're struggling with material, perhaps after an exam, I get them to explain their thinking, and we work together to build a strategy from there. I frequently meet students outside of official office hours if they require one-on-one guidance. If students are interested in topics outside the course I will often extend my office hours.

- Inclusivity

  In the US, access to STEM education is unevenly distributed. This is considered a critical equity issue in education, as high-level math and science courses are often out of reach for students in both rural and urban areas.

  My large lower division courses cater to a wide range of students. They come from different backgrounds, and they enter the class with goals of pursuing a variety of different academic majors. Many of my students have limited mathematical experience, and a sizable minority have had such poor tuition that they are unfamiliar with even the most basic concepts in algebra and geometry. These students are often exceptionally strong academically, and just need the right guidance to succeed.

  To address this gap in experience, I try to encourage my students to explore their foundational knowledge and find out where it is lacking. In class and office hours I frequently ask questions, monitoring their understanding. If there is a concept they are confused about, I always take the time to explain it carefully, avoiding words like "easy" or "simple". I'm acutely aware that for a number of students this may be the first time they've ever had it explained to them. This approach can help transform a student's perception of both the subject and their own place within it.

  In addition to this, I feel it's important to provide extra support outside the class for those who want it. I invite all my students to meet me one-on-one if they need my advice and support. These sessions are often devoted to building strategies to address shortcomings in their prior knowledge. As an example, I often set extra homework questions specifically tailored to the student, designed to highlight an important foundational concept. This personal attention to detail can radically improves student understanding.

- Graduate Student Instructor Support

In large classes a vital role is played by graduate student instructors (GSIs) . It's frequently the case that a student will interact with a GSI more than the main instructor. As such, supporting and managing GSIs is an extremely important task.

At the beginning of a course I provide all GSIs with a comprehensive overview of their duties, from in class expectations to proctoring and grading duties (Appendix B). Throughout the course I stay in constant contact with GSIs, making sure everything is running smoothly and encouraging them to come to me with questions and concerns. This is especially important for first time GSIs who may need extensive guidance. When grading exams I provide GSIs with comprehensive rubric making the process as uniform as possible.

I constantly strive to improve my own teaching, whether it be explaining complex concepts, adapting to the evolving needs of my students, or managing online resources. Ultimately, all of this is in the service of one purpose: to help my students succeed. They are our future doctors, engineers, economists, architects, business leaders, scientists, and mathematicians. It is my privilege to play a part in their academic journey.

Teaching is so much more than standing in front of a class and explaining something you understand and they do not. It is about the most basic human urge - to connect and share a common passion. Done well, it is of lasting significance for everyone involved.